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Abstract

This paper investigates the use of automatic differentiation (AD) as a means for generating sensitivity analyses in rotorcraft design and optimization. This technique transforms an existing computer program into a new program that performs sensitivity analysis in addition to the original analysis. The original FORTRAN program calculates a set of dependent (output) variables from a set of independent (input) variables, the new FORTRAN program calculates the partial derivatives of the dependent variables with respect to the independent variables. The AD technique is a systematic implementation of the chain rule of differentiation, this method produces derivatives to machine accuracy at a cost that is comparable with that of finite-differencing methods. For this study, an analysis code that consists of the Langley-developed hover analysis HOVT, the comprehensive rotor analysis CAMRAD/JA, and associated preprocessors is processed through the AD preprocessor ADIFOR 2.0. The resulting derivatives are compared with derivatives obtained from finite-differencing techniques. The derivatives obtained with ADIFOR 2.0 are exact within machine accuracy and do not depend on the selection of step-size, as are the derivatives obtained with finite-differencing techniques.

Introduction

Over the past 15 years, optimization methods have been increasingly applied to rotorcraft design problems (e.g., Refs. 1–10). Most of these methods use a gradient-based optimizer. An important aspect of any optimization method that uses a gradient-based optimizer is the sensitivity analysis, which is the calculation of derivatives of the objective function and constraints with respect to the design variables. In most rotorcraft optimization applications (e.g., Refs. 1–7), finite-difference techniques are used to calculate the derivatives. Some formulations (e.g., Refs. 8 and 9) have derived analytical

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expressions for the derivatives. Reference 10 uses a semi-analytical approach (a combination of analytical and finite-difference derivatives). Finite-difference derivatives are easy to implement but are step-size dependent and can be costly to compute, particularly if the analysis is computer-intensive. Analytical derivatives are fast to compute and are not step-size dependent. However, derivation of the analytical expressions can be time-consuming. Furthermore, in most comprehensive rotorcraft analyses one does not always have access to analytical expressions for the analysis. Symbolic manipulation methods use the computer to manipulate the analytical expressions and can lead to large cumbersome expressions. An alternate method is the use of automatic differentiation (AD) methods. Although these methods have existed for nearly 30 years, only recently have they been applied to engineering design and optimization problems (Refs. 11–15).

The AD technique transforms an existing computer program into a new program that performs both a sensitivity analysis and the original analysis. If the original FORTRAN program calculates a set of dependent (output) variables from a set of independent (input) variables, then the new FORTRAN program calculates the partial derivatives of the dependent variables with respect to the independent variables. The AD technique is not an automatic implementation of finite differencing that produces approximate derivatives and is dependent upon proper step size, nor is it related to symbolic manipulation, which requires reprogramming in a special-purpose language and results in convoluted expressions for the derivatives. Rather, AD is a systematic implementation of the chain rule of differentiation, it produces derivatives to machine accuracy at a cost that is comparable with that of finite-differencing methods. As pointed out in Reference 11, derivatives calculated with the AD method are faster than finite-difference methods but are not as fast as analytical methods. The AD derivatives, however, are easier to implement than analytical methods.

Recently, progress has been made in developing a general-purpose AD tool called Automatic Differentiation In FORTRAN (ADIFOR), this project is a joint effort between Argonne National Laboratory and Rice University, with funding by the Department of Energy, the National Aeronautics and Space Administration, and the National Science Foundation. The reader is referred to References 11–17 for an overview of the principles that underlie ADIFOR. Work on the ADIFOR project has been underway since 1991. The prototype version, ADIFOR 1.0 (Ref. 17), was in use by June of 1993 and has successfully been used to develop sensitivity derivatives for structural analysis (Ref. 11) and for state-of-the-art computational fluid dynamics (CFD) codes (Refs. 14–16). In Reference 15, ADIFOR 1.0 is used to generate sensitivity derivatives in conjunction with an optimization procedure in the aerodynamic design of a transport type of aircraft.

Recently, an enhanced version, ADIFOR 2.0 (Refs. 18 and 19), has been released. This version, which has the capability to process complex functions, is used in the research presented. Version 2.0 is now available for educational and nonprofit research and for commercial evaluation. Information in regard to ADIFOR 2.0 can be found on the World Wide Web at either of two locations:

http://www.mcs.anl.gov/adifor
or
http://www.cs.rice.edu/~adifor

The objective of the research presented in this paper is to apply AD techniques, in particular ADIFOR 2.0, to typical helicopter analysis codes and to evaluate the resulting sensitivity analysis codes for use in design and optimization. First, the implementation of ADIFOR 2.0 will be discussed, followed by a discussion of the rotorcraft analyses. Finally, derivatives generated by ADIFOR 2.0 will be compared with, on the basis of accuracy and timing, those generated by finite-differencing methods.

General ADIFOR Implementation

To implement ADIFOR 2.0 (see Refs. 18 and 19 for complete instructions), the analysis source code must be available in ANSI standard FORTRAN 77. The code is modified by inserting special lines of code that identify the independent (input) and dependent (output) variables. ADIFOR tracks the dependency between the independent and dependent variables throughout the code. The analysis code is then further modified automatically by the ADIFOR 2.0 preprocessor, which augments the code to calculate partial derivatives. The ADIFOR preprocessor executes on a Sun SPARCstation or IBM RS6000 workstation, however, the augmented (or modified) FORTRAN code generated by ADIFOR can be compiled and executed on any computer. The modified analysis code is compiled conventionally, linked with the ADIFOR library, and executed. ADIFOR provides libraries for Sun SPARCstation and IBM RS6000 computers, and source code for the library routines if execution will be carried out on another computer. Figure 1 shows an example of a simple FORTRAN code before and after ADIFOR processing. As shown in the figure, a single line of code expands to seven lines. When large codes are being used, the effect of this expansion on computer resources must be considered.

Original line of code:

$$hp = omega * q / 550.$$

Augmented code from ADIFOR:

$$r2b = 1.0 / 550.$$
$$r3b = r2b * q$$
$$r4b = r2b * omega$$
$$do \quad i = 1, np$$
$$\nabla hp(i) = r3b * \nabla omega(i) + r4b * \nabla q(i)$$
$$enddo$$
$$hp = omega * q / 550.$$

$\nabla omega(i)$ and $\nabla q(i)$ are vectors containing the derivatives of omega and q with respect to the hp parameters generated upstream in the code. $\nabla hp(i)$ contains derivatives of hp.

Figure 1. Augmented code for one line of code.

Rotorcraft Analysis

To evaluate whether AD techniques can be used in rotorcraft design and optimization, the analysis portion of the optimization procedure described in Reference 4 is used. A summary of the optimization procedure is given here. The optimization procedure minimizes an objective function that is a linear combination of the horsepower required (in hover, forward flight, and maneuver) and vibratory hub shear. The design variables include maximum pretwist, point of taper initiation, taper ratio, root chord, blade stiffnesses, tuning masses, and tuning mass locations. Constraints consist of limits on the horsepower required in hover, forward flight, and maneuver, airfoil section stall, drag divergence Mach number, minimum tip chord, trim, blade natural frequencies, minimum autorotational inertia, and maximum blade weight. The procedure couples the Langley-developed hover analysis HOVT (a strip-theory momentum analysis based on Ref. 20), the comprehensive rotor analysis code CAMRAD/JA (Ref. 21) for forward flight and maneuver, and various processors that translate design variables into appropriate input for the HOVT and CAMRAD/JA codes. (Note: In Reference 4 the HOVT and CAMRAD/JA are implemented as subroutines). HOVT is used to predict the horsepower required in hover, and CAMRAD/JA is used to predict rotor performance, loads, and frequencies. Both HOVT and CAMRAD/JA use tables of experimental two-dimensional airfoil data. The sensitivity analysis consists of finite-difference derivatives of the objective function and constraints. For the work presented here a subset of the design variables(maximum pretwist, point of taper initiation, taper ratio, and root chord) will be used

ADIFOR Implementation in Rotorcraft Analysis

The analysis code described in Reference 4 consists of 367 subroutines. The largest portion of the code is the CAMRAD/JA (326 routines). Initially, CAMRAD/JA was processed with ADIFOR 1.0 (Ref. 17), however, because ADIFOR 1.0 did not support complex numbers, research was postponed until ADIFOR 2.0 became available. In fact, CAMRAD/JA was used by ADIFOR developers in upgrading to ADIFOR 2.0. The CAMRAD/JA code was the largest code processed by ADIFOR 2.0, therefore it was used to test ADIFOR's capabilities. The original, unmodified code in Reference 4 is approximately 83,500 lines. After it was processed through ADIFOR 2.0, the modified code which included the analysis and sensitivity codes was 197,908 lines.

A few modifications to the analysis code were necessary because ADIFOR 2.0 requires that the code be in FORTRAN 77. Because ADIFOR 2.0 does not support NAMELIST, the most time-consuming modification was commenting out the 280 references to NAMELIST as well as the associated input and output references to NAMELIST, during processing through ADIFOR 2.0. These statements are reinstated during actual execution of the modified code. In addition, the source codes of all math library routines had to be included in the analysis code. Although ADIFOR 2.0 will process even if the source codes for the math library routines are not included, the resulting derivative may be incorrect because a portion of the derivative may not have been calculated. For example, the HOVT code uses a math library code to interpolate the airfoil table information, and the source code for the math library was not included in the original analysis processed through ADIFOR 2.0. The resulting derivative was incomplete because the dependency on the information in the airfoil tables was not taken into account. After the source code for the math library code was included in the original code and processed through ADIFOR, the resulting derivative was complete.

Results

Because the purpose of this paper is to evaluate the use of AD techniques to generate sensitivity derivatives in rotorcraft analyses, ADIFOR 2.0 is applied in a "black box" manner without any clever implementation as was done in Reference 14. The entire analysis code described in Reference 4 is processed through ADIFOR 2.0. Only a subset of the design variables and response quantities described in Reference 4 is used here. The research is done in two phases. In the first phase, ADIFOR 2.0 is applied to the hover analysis code HOVT and the appropriate preprocessors. The HOVT code is fairly simple in comparison with the CAMRAD/JA code, furthermore, the processing through ADIFOR 2.0 is easier to understand. The lessons learned in this first phase are applied in the second phase. In the second phase, ADIFOR 2.0 is applied to CAMARD/JA and the appropriate preprocessors.

Of the design variables used in Reference 4, only four affect the HOVT analysis. The main output from HOVT is the hover horsepower required. Thus, four independent (input) variables and one dependent (output) variable are used. The four independent variables, shown in Figure 2, are maximum pretwist, point of taper initiation, taper ratio, and blade root chord. The blade planform is rectangular from the root to the point of taper initiation and then tapers linearly to the tip. The blade pretwist varies linearly from the root to its maximum value at the tip. The output variable is hover horsepower required. Derivatives are generated with the nominal values and flight conditions shown in Table 1.

Table 2 compares the derivatives of the hover horsepower with respect to each independent variable obtained with ADIFOR 2.0, the finite-difference derivatives were obtained with a step size of 0.00001. (Note this step size was determined to be the best for generating finite-difference derivatives after a step size study in Ref. 4.) Both sets of derivatives compare well. However, the finite-difference derivative is step-size dependent, the ADIFOR derivative is not. Figure 3 shows the finite-difference derivative of the hover horsepower required with respect to pretwist as a function of step size. The ADIFOR-generated derivative is shown as a straight line since it is an exact derivative (within machine accuracy). For small step sizes (i.e., less than 0.0001), the ADIFOR derivatives and the finite-difference derivatives are comparable. As the step size increases, the two sets of derivatives do not agree as well because the accuracy of the finite-difference derivative is in question. The figure shows one area (for a step size greater than 0.0006) in which the results do not agree well. In fact, the finite-difference derivative has a different sign. This result exemplifies the caution that should be exercised in using finite-difference derivatives. Although not included in this paper, similar comparisons resulted for the other three input variables.

In the second phase, ADIFOR 2.0 is applied to the comprehensive rotor analysis code, CAMRAD/JA and associated preprocessors. The same four independent variables are used. The four output variables are required horsepower and hub shear for both forward flight and maneuver. Table 3 compares the derivatives for the required forward-light horsepower and hub shear obtained from ADIFOR 2.0 with those obtained from finite differencing for two step sizes. For a step size of 0.00001, the ADIFOR derivatives and the finite-difference derivatives agree well. For a larger step size of 0.0001, only the derivatives with respect to taper ratio and root chord agree. Figure 4 shows the step-size dependency for the finite-difference derivatives of required horsepower with respect to pretwist as a function of step size.

Table 4 compares the derivatives of hub shear with respect to the four input variables for the forward-flight condition. The finite-difference derivatives are highly step size dependent. For the small step size (0.00001), the finite-difference derivatives agree well with the ADIFOR derivatives, only the results for the derivative with respect to point of taper initiation do not agree well. Figure 5 shows that the finite-difference derivative for hub shear with respect to pretwist is a function of step size while the ADIFOR derivative is not.

Table 5 compares the derivatives of the horsepower required for maneuver with respect to the four input variables. For the small step size (0.00001), the derivatives agree well. For the larger step size, the derivatives with respect to taper ratio and root chord agree well. The agreement for the derivatives for the other two input variables is not as good. Figure 6 shows the finite-difference derivative of horsepower with respect to pretwist as a function of step size, the ADIFOR derivative is not a function of step size.

Table 6 compares derivatives of hub shear with respect to the four input variables for the maneuver flight condition. These results are similar to those previously discussed. For both step sizes, the derivatives agree well. Figure 6 shows the derivative of the maneuver hub shear as a function of step size.

The results presented in Figures 3 through 6 and Tables 2 through 6 demonstrate the dependence of finite-difference derivatives on step size; a step size that is effective for one input variable and one dependent variable is not necessarily effective for a different dependent variable. For example, the finite-difference derivatives for the required hover horsepower, required forward flight horsepower, and the required maneuver horsepower all have different step sizes that lead to effective finite-difference derivatives. Also note that for finite-difference derivatives obtained with the CAMRAD/JA code a range of step sizes exists between 0.00005 and 0.0001 in which the finite-difference derivatives are unacceptable. The derivatives generated by ADIFOR 2.0 do not have this disadvantage.

Although it was anticipated that ADIFOR would be a faster method for obtaining partial derivatives than using the finite differencing method, the opposite occurred. To calculate the 4 derivatives, the ADIFOR method was approximately 16 percent slower. In the CAMRAD/JA trim, three nested loops are present, ADIFOR 2.0 generates code that must be evaluated at each step in these loops. This process increases the time that is required to evaluate the ADIFOR-generated derivatives. Clever implementation of ADIFOR (see Ref. 14) which did not process such trim loops may possibly decrease the time required to evaluate the ADIFOR-generated derivatives. On the other hand, the ADIFOR derivatives are accurate, unlike those generated with finite differencing which are dependent on step size. The time spent in identification of the best step size can eliminate any difference in time between the two methods. It took approximately 2 weeks to process the code through ADIFOR 2.0.

Conclusions

Rotorcraft design and optimization require sensitivity information. For some rotorcraft analyses, analytical techniques exist to generate those sensitivities. For most rotorcraft analyses, however, those derivatives have not been developed. Automatic differentiation (AD) is a viable alternative for generating those sensitivities. Exact derivatives are generated at a cost that is comparable with that required for the step-size-dependent finite-difference derivatives. This paper evaluates whether automatic differentiation techniques, in particular ADIFOR 2.0, can be used to generate sensitivity derivatives for rotorcraft analyses. The ADIFOR 2.0 preprocessor was successfully used to generate sensitivity derivatives for the Langley-developed hover code HOVT and the comprehensive rotorcraft analysis CAMRAD/JA. To date this research is the largest code to which ADIFOR 2.0 has been implemented.

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Table 1. Parameters and Flight Conditions

Parameters			
Maximum pretwist	–16 degrees		
Point of taper initiation	0.8 R		
Taper ratio	3.0		
Blade root chord	0.44976 ft		
Blade radius R	4.685 ft		
Flight Conditions			
Rotational velocity	639.5 RPM (in Freon density of 0.006 slug/ft^3)		
Hover tip Mach number	0.628		
	Hover	Forward flight	Maneuver
C_L	0.00810	0.00810	0.00985
C_D	-	–0.000811	–0.000596
Advance ratio	-	0.35	0.30

Table 2. Derivatives of Hover Horsepower with Respect to Input Variables

Independent variable	ADIFOR derivatives	Finite-difference derivatives
Pretwist, degree	–0.025586555 hp/deg	–0.025570612 hp/deg
Taper initiation point	1.5392865 hp	1.5392717 hp
Taper ratio	–0.02634087 hp	–0.0262997 hp
Root chord, ft	4.5522245 hp/ft	4.5520691 hp/ft

Table 3. Derivatives of Forward-Flight Horsepower with Respect to Input Variables

Independent variable	ADIFOR derivatives	Finite-differences derivatives step size = 0.00001	Finite-difference derivatives step size = 0.0001
Pretwist, degree	–0.114343 hp/deg	–.113255 hp/deg	–.836424 hp/deg
Taper initiation point	3.617647 hp	2.818897 hp	17.913394 hp
Taper ratio	–0.164071 hp	–0.164253 hp	–0.164099 hp
Root chord	10.005027 hp/ft	10.006285 hp/ft	10.004409 hp/ft

Table 4. Derivatives of Forward-Flight Hub Shear with Respect to Input Variables

Independent variable	ADIFOR derivatives	Finite-difference derivatives step size = 0.00001	Finite-difference derivatives step size = 0.0001
Pretwist, degree	–0.025312 lb/deg	–0.025015 lb/deg	–0.91453 lb/deg
Taper initiation point	–0.68264 lb	0.58847 lb	17.513 lb
Taper ratio	–0.054618 lb	–0.051294 lb	–0.054967 lb
Root chord, ft	–1.4649 lb/ft	–1.4347 lb/ft	–1.4614 lb/ft

Table 5. Derivatives of Maneuver Horsepower with Respect to Input Variables

Independent variable	ADIFOR derivatives	Finite–difference derivatives step size = 0.00001	Finite–difference derivatives step size = 0.0001
Pretwist, degree	1.374133 hp/deg	1.371376 hp/deg	0.6989315 hp/deg
Taper initiation point	–145.9513 hp	–144.3115 hp	–128.8663 hp
Taper ratio	4.228978 hp	4.29695 hp	4.231664 hp
Root chord, ft	–366.2637 hp/ft	–366.0956 hp/ft	–365.9497 hp/ft

Table 6. Derivatives of Maneuver Hub Shear with Respect to Input Variables

Independent variable	ADIFOR derivatives	Finite–difference derivatives step size = 0.00001	Finite–difference derivatives step size = 0.0001
Pretwist, degree	0.17865 lb/deg	0.17927 lb/deg	0.21010 lb/deg
Taper initiation point	–20.074 lb	–20.775 lb	–21.461 lb
Taper ratio	0.594734 lb	0.555863 lb	0.59128 lb
Root chord, ft	–53.919 lb/ft	–54.172 lb/ft	–53.845 lb/ft

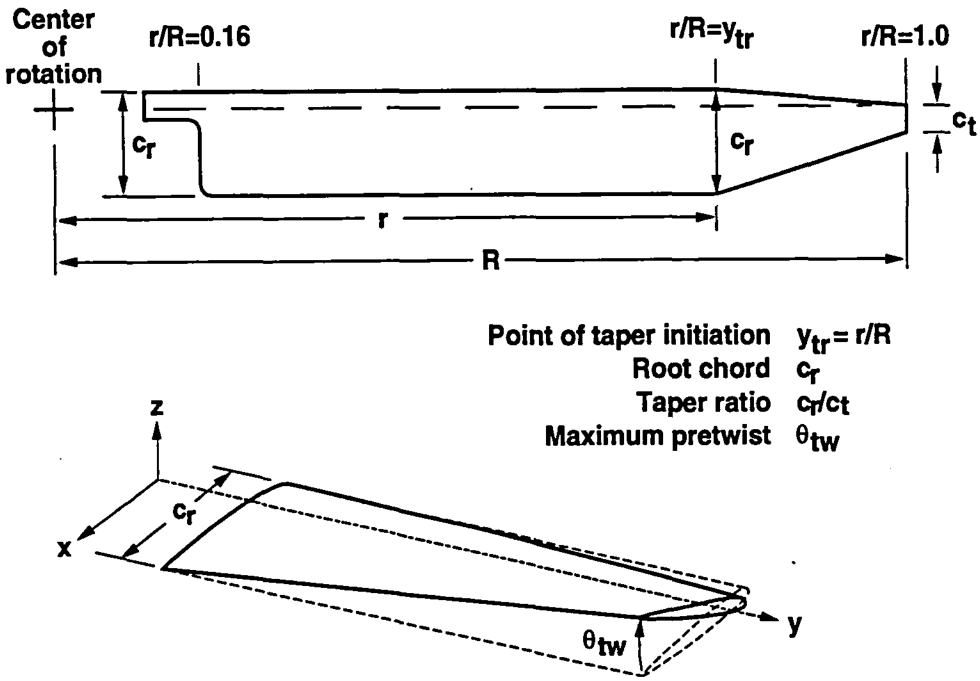

Figure 2. Four independent variables.

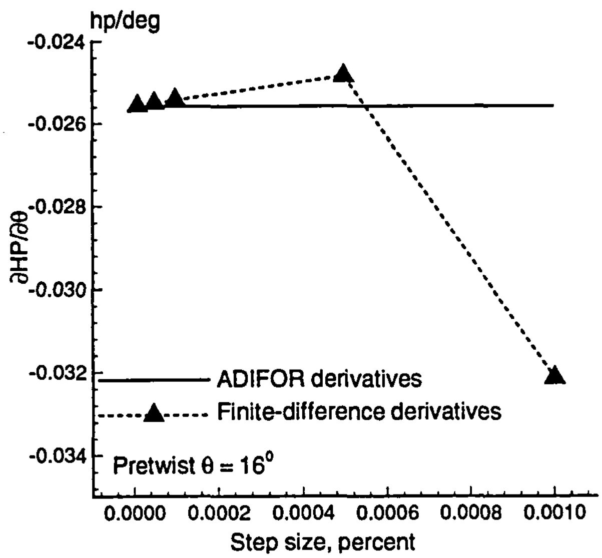

Figure 3. Hover horsepower derivatives.

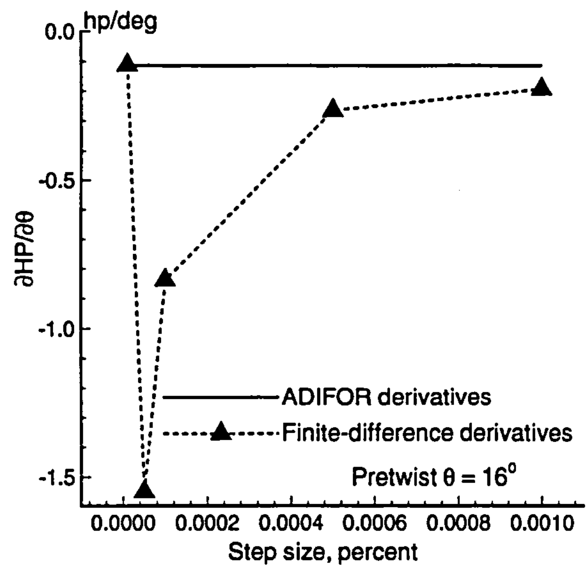

Figure 4. Forward-flight horsepower derivatives.

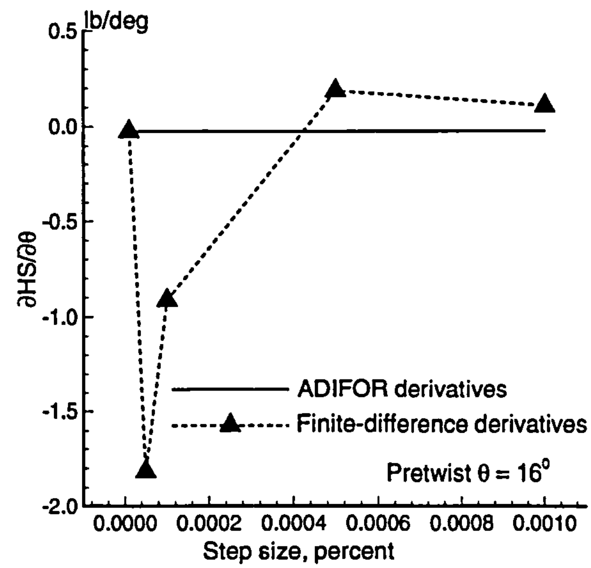

Figure 5. Forward-flight hub shear derivatives.

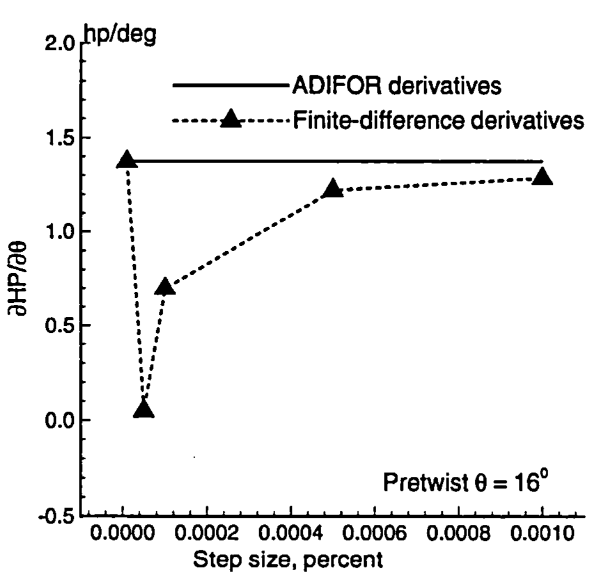

Figure 6. Maneuver horsepower derivatives.

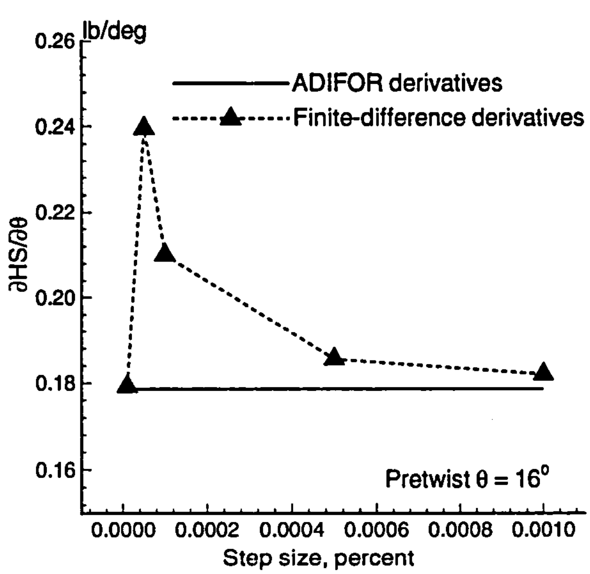

Figure 7. Maneuver hub shear derivatives.